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DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

NOTE.—Thomas S. Clarkson remarks that the solution of 321 does not satisfy the conditions of the problem, since the whole number of shares is not to exceed 200. He contends that the problem is impossible as stated, and suggests that it may have been intended that each man's share is not to exceed 200. We think the problem is stated as intended and is therefore impossible. ED. F.

325. Proposed by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

I have a chronometer whose rate is uniform. When it indicates t_1 time at Washington I find that it is h_1 hours slow. I take it to Philadelphia and when it indicates t_2 time, the local time of Philadelphia is h_2 hours faster. I bring my chronometer back to Washington and find that when it indicates t_3 time it is h_3 hours slow. If $t_1=5$ A. M., $t_2=7$ hours, 54 minutes, $t_3=11$ hours, 46 minutes A. M., $h_1=1$ hour, $h_2=1$ 203/900 hours, $h_3=1$ 7/30 hours, find the difference of longitude between Washington and Philadelphia.

Solution by J. EDWARD SANDERS, Weather Bureau, Chicago, Ill, and the PROPOSER.

$(h_3-h_1)/(t_3-t_1)$=error for one hour.

$(t_2-t_1)(h_3-h_1)/(t_3-t_1)$=error at time t_2 in Philadelphia.

$\therefore h_1+t_2+(t_2-t_1)(h_3-h_1)/(t_3-t_1)$=time at Washington.

h_2+t_2=time at Philadelphia.

$h_2+t_2-h_1-t_2-(t_2-t_1)(h_3-h_1)/(t_3-t_1)$=difference of time between the two cities=T.

$$\therefore T=\frac{h_2(t_3-t_1)+h_3(t_1-t_2)+h_1(t_2-t_3)}{t_3-t_1}.$$

$15T=L$=difference in longitude.

$$\therefore L=\frac{15(h_1t_2+h_2t_3+h_3t_1-h_1t_3-h_2t_1-h_3t_2)}{t_3-t_1}.$$

Putting $h_1=1$ hour, $h_2=1\frac{203}{900}$ hours, $h_3=1\frac{7}{30}$ hours, $t_1=5$ hours, $t_2=$ 7 hours 54 minutes, $t_3=11$ hours 46 minutes, we get

$$L=\tfrac{113}{60}^\circ=1^\circ\ 53'.$$

Also solved by T. S. Clarkson.

326. Proposed by R. D. CARMICHAEL, Princeton University.

Is the series, of which the nth term is $\dfrac{1.3.5.7\ldots(2n-1)}{(n+1)!\,2^n.(2n+3)}$ convergent? If so, find its sum.

Solution by E. B. ESCOTT, Ann Arbor, Mich.

Apply the test given in Hall and Knight's *Higher Algebra*, 3rd ed., p. 244.

If $\underset{n \doteq \infty}{\text{Lim}} \left[n \left(\frac{u_n}{u_{n+1}} - 1 \right) \right] > 1$, the series is convergent. In this case

$$\frac{u_n}{u_{n+1}} = \frac{(n+2)2(2n+5)}{(2n+1)(2n+3)}.$$

The above limit$=\frac{5}{2}$. Therefore, the series is convergent.

To find its value, we may use the following method due to Euler:

$$\text{Let } y = \ldots + \frac{1.3.5\ldots(2n-1)}{(n+1)!\,2^n\,(2n+3)} x^{2n+3} + \frac{1.3.5\ldots(2n+1)}{(n+2)!\,2^{n+1}(2n+5)} x^{2n+5} + \ldots$$

$$\frac{dy}{dx} = \ldots + \frac{1.3.5\ldots(2n-1)}{(n+1)!2^n} x^{2n+2} + \frac{1.3.5\ldots(2n+1)}{(n+2)!2^{n+1}} x^{2n+4} + \ldots$$

$$\frac{d^2y}{dx^2} = \ldots + \frac{1.3.5\ldots(2n-1)}{n!2^{n-1}} x^{2n+1} + \frac{1.3.5\ldots(2n+1)}{(n+1)!2^n} x^{2n+3} + \ldots$$

Divide by x^3 and integrate,

$$\int \frac{1}{x^3} \frac{d^2y}{dx^2} dx = \ldots + \frac{1.3.5\ldots(2n-3)}{n!\;2^{n-1}} x^{2n-1} + \frac{1.3.5\ldots(2n-1)}{(n+1)!2^n} x^{2n+1} + \ldots$$

Multiply by x and integrating again,

$$\int x \int \frac{1}{x^3} \frac{d^2y}{dx^2} dx\, dx = \ldots + \frac{1.3.5\ldots(2n-3)}{n!\;2^{n-1}(2n+1)} x^{2n+1} + \frac{1.3.5\ldots(2n-1)}{(n+1)!\,2^n\,(2n+3)} x^{2n+3}$$

$$+ \ldots = y.$$

Then differentiating, dividing by x, and differentiating again, we have

$$\frac{1}{x^3} \frac{d^2y}{dx^2} = \frac{1}{x} \frac{d^2y}{dx^2} - \frac{1}{x^2} \frac{dy}{dx}.$$

This differential equation may be written

$$(1-x^2)\frac{d^2y}{dx^2}+x\frac{dy}{dx}=0.$$

Solving the differential equation, we have

$$y=\frac{c}{2}[x\sqrt{(1-x^2)}+\sin^{-1}x].$$

To verify that this gives the given series, we have by differentiating,

$$\frac{dy}{dx}=c\sqrt{(1-x^2)}=c(1-\frac{1}{2}x^2-\frac{1}{2!\,2^2}x^4-\frac{1.3}{3!\,2^3}x^6-\dots$$
$$-\frac{1.3.5\dots(2n-1)}{(n+1)!\,2^{n+1}}x^{2n+2}-\dots).$$

Integrating, $y=c(x-\frac{1}{2.3}x^3-\frac{1}{2!2^2.5}x^5-\frac{1.3}{3!2^3.7}x^7-\dots$

$$-\frac{1.3.5\dots(2n-1)}{(n+1)!\,2^{n+1}(2n+3)}x^{2n+3}-\dots).$$

Putting $c=-2$, we have

$$\lim_{x\doteq 1}[-(x\sqrt{(1-x^2)}+\sin^{-1}x)]=-\frac{\pi}{2}=-2+\frac{1}{3}+\frac{1}{2!\,2.5}+\frac{1.3}{3!2^2.7}+\dots$$

Also solved by G. B. M. Zerr, who found for the limit of the sum of n terms of the series as $n\doteq\infty$, $\frac{5}{3}-\frac{1}{2}\pi$.

GEOMETRY.

349. Proposed by J. A. CAPARO, Notre Dame University, Notre Dame, Indiana.

Given the radius of a circular smooth cylinder and its position with respect to a source of light and the eye. Find a geometrical construction to determine the line of brilliancy.

Solution by C. N. SCHMALL, New York City.

By a simple examination of the data* it is clear that the problem reduces to the following: Given a circle and two points A, B, outside (in same plane), to find a point P on the circumference, such that AP and

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*See *Encyclopedia Britannica*, Vol. XIV, page 589.